TCS 2018 – bonus assignment series 2

Deadline for handing in: March 19, 2018 at 23:59 hr

This assignment may be made individually or (preferably!) by pairs of students. Don't forget to mention your name(s) in the assignment. You can hand me over the results on paper or put them in my post box (BOU 8-10, secretariat), or alternatively send the results by email (**as a single PDF file!**) to: uiterwijk@maastrichtuniversity.nl.

If accessed as sufficient, you earn 0.33 bonus point to your final mark for TCS.

1) Show a context-free grammar for each of the following languages L:
   a) $\{a^i b^j : j = 3i + 3\}$.
   b) $\{a^m b^n : m \geq n, m\text{-}n \text{ is odd}\}$.
   c) $\{a^i b^j c^k, i > k, 0 \leq j < 3, k \geq 0\}$.

2) Let L be the language generated by the following grammar G:

   $S \rightarrow 1S2$
   $S \rightarrow 1S2S$
   $S \rightarrow 12$

   a) Show the first five strings in a lexicographic enumeration of L.
   b) Prove that L is not regular.

3) Build a PDA to accept each of the following languages L:
   a) $\{a^i b^j : j = 4i + 2\}$.
   b) $\{a^i b^j c^k, i > k, 0 \leq j < 3, k \geq 0\}$
   c) $\{a^m b^{2n} c^{3n} d^p : p > m, \text{ and } m, n \geq 1\}$.

4) Proof by pumping that the following language is not context-free:

   $L = \{a^n b^m a^n b^m a^n : n, m \geq 0\}$.

5) Let $L = \{w \in \{a, \#\}^* : w \text{ is in the sequence: } a, a\#aa, a\#aa\#aaa, a\#aa\#aaa\#aaaa, \ldots\}$.
   a) Is L regular, context-free but not regular, or not context-free? Prove your answer.
   b) Now consider $\neg L$. Is it regular, context-free but not regular, or not context-free? Prove your answer.