Universiteit Maastricht
Faculty of Humanities and Sciences
Knowledge Engineering

Exam

Logic

Wednesday 30 May 2007

This exam is NOT an open-book exam.
You will get an auxiliary sheet with the most important formulas.

Clarify all your answers sufficiently!
Indicate in semantic tableaux exactly which reduction rules you are using.
In the case of derivations, indicate exactly from which formula(s) each formula is derived and if the conditions are met (if applicable).

This exam contains 9 assignments. For each assignment the maximum score that you can obtain for a correct answer is indicated. The final score for the exam is the sum of the scores for the individual assignments, divided by 10.

1. (10 points).
   Inspector Craig is confronted with a nasty case, in which there are four suspects: Albert, Bernard, Colin, and Dave. Craig has established the following four facts:
   i. If Albert is guilty, then Bernard is guilty too.
   ii. If Bernard is guilty, then Colin is guilty or Albert is innocent.
   iii. If Dave is innocent, then Albert is guilty and Colin is innocent.
   iv. If Dave is guilty, then so is Albert.

   Which of the suspects are certainly guilty?
   Determine the answer with the aid of a truth table (model elimination).

2. Semantic tableaux propositional logic (12 points).
   a. Explain the $\rightarrow_R$ reduction rule.
   b. Given is the consequence $p \rightarrow q, (q \rightarrow p) \rightarrow \neg r \;/\; q \vee r$. Investigate the validity of this consequence with the aid of a semantic tableau. If this consequence is not valid, give all the counterexamples.
   c. Investigate by means of a semantic tableau whether the set of formulas $\{(p \vee q) \rightarrow r, p \leftrightarrow \neg q, \neg r\}$ is consistent or inconsistent. If the set is consistent, then give a valuation that is a model of the set.

3. Natural deduction propositional logic (12 points).
   Prove by means of natural deduction:
   a. $p \rightarrow q, (q \wedge r) \rightarrow s \vdash (p \wedge r) \rightarrow s$
   b. $(p \vee s) \rightarrow (q \vee r), (q \vee s) \rightarrow r \vdash p \rightarrow r$
   c. $(p \wedge q) \rightarrow \neg r, r \rightarrow q \vdash \neg p \vee \neg r$
      (hint: use *reductio ad absurdum*, i.e., assume the negation of $\neg p \vee \neg r$)

4. (10 points).
   Translate the following formula into an equivalent formula in prenex form:
   $\forall x \exists z(\forall y \mathrm{P}xyz \rightarrow ((\forall z \mathrm{Q}xz \wedge \mathrm{R}z) \vee \neg(\forall x \mathrm{R}x \wedge \exists y \mathrm{Q}xy)))$

5. (12 points).
   Consider the domain of people. Some people are prophets, who may have followers. Px means that x is a prophet. Fxy means that x is a follower of y. The predicate = may also be used. Now write each of the following sentences as a formula of predicate logic:
   a. All prophets have at least one follower.
   b. There is a prophet who has precisely two followers.
   c. A follower of a prophet also follows any prophet that the first prophet follows.

6. Semantic tableaux predicate logic (12 points).
   a. Explain the $\exists_R$ reduction rule.
   Investigate the validity of the following consequences by means of a semantic tableau. If a consequence is not valid, give at least one counterexample.
   b. $\forall x(\mathrm{A}x \rightarrow \mathrm{B}x), \neg\exists x\mathrm{B}x \,/\, \neg\exists x\mathrm{A}x$
   c. $\forall x(\mathrm{A}x \vee \mathrm{B}x), \exists x(\mathrm{C}x \rightarrow \mathrm{A}x), \exists x\mathrm{B}x \,/\, \exists x\mathrm{A}x$

7. Natural deduction predicate logic (12 points).
   a. Explain the $\forall$I rule. Why is the condition essential?
   Prove by means of natural deduction:
   b. $\neg\exists x\mathrm{A}x \vdash \forall x\neg\mathrm{A}x$
   c. $\exists x(\mathrm{A}x \vee \neg\mathrm{B}x), \forall x(\mathrm{A}x \rightarrow \mathrm{C}x), \forall x(\neg\mathrm{C}x \rightarrow \mathrm{B}x) \vdash \exists x\mathrm{C}x$

8. (10 points).
   Consider the collection of those formulas from the proposition logic which do not contain the connective $\neg$ (thus only the connectives $\wedge$, $\vee$, $\rightarrow$, and $\leftrightarrow$ occur). Let Σ_ψ be the set of proposition letters that occur in formula ψ. Prove with the aid of formula induction that for all formulas φ holds: $\Sigma_\varphi \models \varphi$.

9. (10 points).
   Given are models $M_1 = (\boldsymbol{D}, I_1)$ and $M_2 = (\boldsymbol{D}, I_2)$.
   Structure $\boldsymbol{D} = \langle \mathbb{R}, <, 0, 1, *, + \rangle$. The interpretation functions are defined as $I_1(R)=I_2(R)=<$, $I_1(a)=I_2(b)=0$, $I_1(b)=I_2(a)=1$, $I_1(f)=I_2(f)=*$, and $I_1(g)=I_2(g)=+$.
   a. Given the formula $\varphi = \forall y R(g(f(f(x,x),x),b), f(y,g(y,a))))$. For which look-up table(s) is this formula true in M_1?
   b. Find a formula φ such that M_1 is a model of φ and M_2 is not a model of φ.